Applied Mathematics

A Comprehensive Course for Leaving Certificate 3rd Edition

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Errata for the first print run (2022)

Errata

Chapter 4 Answers (p.286):
Exercise 4A: Questions 16 & 17 are misnumbered as 14 & 15.
Exercise 4B: Q16(i) first answer should be 12.6°, not 14.9°.

Chapter 5
Example 5A2 (p. 78): final answer should be 57.9 m s^{-1}, not 18.3 m s^{-1}.
Example 5B5 (p.82): the solutions for parts (iv) and (v) are the wrong way around.

Chapter 6
Exercise 6E (p.108): Q15(i) should say before instead of after.

Chapter 6 Answers (p.287):
Exercise 6E: Q13 answers should be: 3.8 m s^{-1} @ 58° N of E, 2.1 m s^{-1} @ 62° N of W.

Chapter 7

Exercise 7D (p.128): Q8(ii) should be $\omega = \sqrt{\dfrac{g}{h}}$.

Extra question – Q11
(i) Derive an expression for the work done when a spring of elastic constant k N m^{-1} is stretched by x m.
(ii) A particle of mass 5 kg is attached to one end of a bungee cord of natural length 2 m and elastic constant 25 N m^{-1}. The other end of the bungee cord is attached to a point Q. The 5 kg particle is held next to point Q, released and allowed to fall vertically. Using conservation of energy, find the extension of the bungee cord when the 5 kg particle next comes to rest.

Chapter 7 Answers (p.288):

Exercise 7D: Q9(ii) answer should be: $\dfrac{g}{2}$ m s^{-2}.

Q11) (i) $\frac{1}{2}kx^2$, (ii) 5.38 m.

Chapter 12
Example 12C2 (p.209)
The alternative solution to the right has been added to this example.

Alternative solution: use $u_n = C\left(k^n\right) + D$

$2 = C\left(3^1\right) + D, \qquad \Rightarrow 2 = 3C + D \qquad \mathbf{C}$

$6 = C\left(3^2\right) + B, \qquad \Rightarrow 6 = 9C + D \qquad \mathbf{D}$

$\mathbf{D} - \mathbf{C}$: $4 = 6C, \qquad \Rightarrow C = \frac{2}{3}$ and then $D = 0$

$\Rightarrow$ the solution is: $u_n = \frac{2}{3}\left(3^n\right) = 2\left(3^{n-1}\right)$

It is a matter of preference whether to use $n-1$ or n as the index in the general solution.

There are calculation errors in examples 12F2, 12F3 & 12F4.

Example 12F2 (p.219)

(a) Solve the inhomogeneous first order difference equation $u_{n+1} = n^2 - 2n + 1 - 3u_n$ with $u_1 = 0$.
(b) Hence find u_9.

Solution: (a) Rewrite it with all terms of form u_n on the left: $u_{n+1} + 3u_n = n^2 - 2n + 1$
The particular solution is of the form: $u_n = a + bn + cn^2$.
The general solution of the difference equation is of the form: $u_n = A(-3)^{n-1}$.
Putting these together gives: $u_n = A(-3)^{n-1} + a + bn + cn^2$.
As there are four unknowns still here, A, a, b and c, we will need the first four terms of the sequence.

$u_1 = 0$ and $u_{n+1} = n^2 - 2n + 1 - 3u_n$

$\Rightarrow u_2 = 1^2 - 2(1) + 1 - 3(0) = 0$

& $u_3 = 2^2 - 2(2) + 1 - 3(0) = 1$

& $u_4 = 3^2 - 2(3) + 1 - 3(1) = 1$

Now use these in our solution to find the four unknowns.

$u_1 = A(-3)^{1-1} + a + b(1) + c(1)^2 = 0,$ $\quad \Rightarrow A + a + b + c = 0$ **A**

$u_2 = A(-3)^{2-1} + a + b(2) + c(2)^2 = 0,$ $\quad \Rightarrow -3A + a + 2b + 4c = 0$ **B**

$u_3 = A(-3)^{3-1} + a + b(3) + c(3)^2 = 1,$ $\quad \Rightarrow 9A + a + 3b + 9c = 1$ **C**

$u_4 = A(-3)^{4-1} + a + b(4) + c(4)^2 = 1,$ $\quad \Rightarrow -27A + a + 4b + 16c = 1$ **D**

$(-9 \times \mathbf{A}) + \mathbf{C}$: $-8a - 6b = 1,$ $\quad \Rightarrow 8a + 6b = -1$ **E**

$(3 \times \mathbf{B}) + \mathbf{C}$: $4a + 9b + 21c = 1$ **F**

$(3 \times \mathbf{C}) + \mathbf{D}$: $4a + 13b + 43c = 4$ **G**

$(43 \times \mathbf{F}) + (-21 \times \mathbf{G})$: $88a + 114b = -41$ **H**

$(-11 \times \mathbf{E}) + \mathbf{H}$: $48b = -30,$ $\quad \Rightarrow b = -\frac{5}{8}$

$\Rightarrow$ in **E**: $8a + 6\left(\frac{-5}{8}\right) = -1,$ $\quad \Rightarrow a = \frac{11}{32}$

$\Rightarrow$ in **F**: $4\left(\frac{11}{32}\right) + 9\left(\frac{-5}{8}\right) + 21c = 1,$ $\quad \Rightarrow c = \frac{1}{4}$

$\Rightarrow$ in **A**: $A + \frac{11}{32} - \frac{5}{8} + \frac{1}{4} = 0,$ $\quad \Rightarrow A = \frac{1}{32}$

$$\Rightarrow u_n = \frac{1}{32}(-3)^{n-1} + \frac{11}{32} - \frac{5n}{8} + \frac{n^2}{4}.$$

(b) $u_9 = \frac{1}{32}(-3)^{9-1} + \frac{11}{32} - \frac{5(9)}{8} + \frac{9^2}{4} = 220$

Example 12F3 (p.220)

(a) Solve the inhomogeneous second order difference equation $u_{n+1} - 3u_n - 4u_{n-1} = 1 - 4n$ with $u_1 = 1$ and $u_2 = 2$.

(b) Hence find u_{10}.

Solution: (a) From the table above the particular solution is of the form: $u_n = a + bn$.

The characteristic equation for the corresponding inhomogeneous difference equation is: $x^2 - 3x - 4 = 0$.

Solving this: $(x+1)(x-4) = 0,$ $\quad \Rightarrow x = -1$ or 4

The general solution of the difference equation is of the form: $u_n = A(-1)^n + B(4)^n$.

Putting these together gives: $u_n = A(-1)^n + B(4)^n + a + bn.$

As there are four unknowns still here, *A*, *B*, *a* and *b*, we will need the first four terms of the sequence.

$u_1 = 1,\ u_2 = 2$ and $u_{n+1} - 3u_n - 4u_{n-1} = 1 - 3n$

$\Rightarrow u_3 - 3(2) - 4(1) = 1 - 3(2),$ $\quad \Rightarrow u_3 = 5$

& $u_4 - 3(5) - 4(2) = 1 - 3(3),$ $\quad \Rightarrow u_4 = 15$

Now use these in our solution to find the four unknowns.

$u_1 = A(-1)^1 + B(4)^1 + a + b(1) = 1, \qquad \Rightarrow -A + 4B + a + b = 1$ **A**

$u_2 = A(-1)^2 + B(4)^2 + a + b(2) = 2, \qquad \Rightarrow A + 16B + a + 2b = 2$ **B**

$u_3 = A(-1)^3 + B(4)^3 + a + b(3) = 5, \qquad \Rightarrow -A + 64B + a + 3b = 5$ **C**

$u_4 = A(-1)^4 + B(4)^4 + a + b(4) = 15, \qquad \Rightarrow A + 256B + a + 4b = 15$ **D**

$\mathbf{D} - \mathbf{B}$: $240B + 2b = 13$ **E**

$\mathbf{C} - \mathbf{A}$: $60B + 2b = 4$ **F**

$\mathbf{E} - \mathbf{F}$: $180B = 9, \qquad \Rightarrow B = \frac{1}{20} = 0.05$

$\Rightarrow$ in **E**: $240\left(\frac{1}{20}\right) + 2b = 13, \qquad \Rightarrow b = \frac{1}{2}$

$\Rightarrow$ in **A**: $-A + \frac{1}{5} + a + \frac{1}{2} = 1, \qquad \Rightarrow -A + a = \frac{3}{10}$ **G**

$\Rightarrow$ in **B**: $A + \frac{4}{5} + a + 1 = 2, \qquad \Rightarrow A + a = \frac{1}{5}$ **H**

$\mathbf{G} + \mathbf{H}$: $2a = 0.5, \qquad \Rightarrow a = 0.25$

$\Rightarrow$ in **H**: $A + 0.25 = \frac{1}{5}, \qquad \Rightarrow A = -0.05, \qquad \Rightarrow u_n = -0.05(-1)^n + 0.05(4)^n + 0.25 + 0.5n.$

(b) $u_{10} = -0.05(-1)^{10} + 0.05(4)^{10} + 0.25 + 0.5(10) = 52434.$

Example 12F4 (p.221)

(a) Solve the difference equation $u_{n+2} = u_{n+1} + 6u_n + 2^n$ with $u_0 = 2$ and $u_1 = 3$.

(b) Hence find u_{10}.

Solution: (a) First of all need to have the difference equation in the right form: $u_{n+2} - u_{n+1} - 6u_n = 2^n$

The particular solution is of the form: $u_n = a + b.2^n$.

The characteristic equation for the corresponding inhomogeneous difference equation is: $x^2 - x - 6 = 0$.

Solving this: $(x+2)(x-3) = 0, \qquad \Rightarrow x = -2$ or 3

The general solution of the difference equation is of the form: $u_n = A(-2)^n + B(3)^n$.

Putting these together gives: $u_n = A(-2)^n + B(3)^n + a + b.2^n$

As there are four unknowns still here, *A*, *B*, *a* and *b*, we will need the first four terms of the sequence.

$u_0 = 2,\ u_1 = 3$ and $u_{n+2} = u_{n+1} + 6u_n + 2^n$

$\Rightarrow u_2 = 3 + 6(2) + 2^0, \qquad \Rightarrow u_2 = 16$

$\&\ u_3 = 16 + 6(3) + 2^1, \qquad \Rightarrow u_3 = 36$

Now use these in our solution to find the three unknowns.

$u_0 = A(-2)^0 + B(3)^0 + a + b.2^0 = 2, \qquad \Rightarrow A + B + a + b = 2$ **A**

$u_1 = A(-2)^1 + B(3)^1 + a + b.2^1 = 3, \qquad \Rightarrow -2A + 3B + a + 2b = 3$ **B**

$u_2 = A(-2)^2 + B(3)^2 + a + b.2^2 = 16, \qquad \Rightarrow 4A + 9B + a + 4b = 16$ **C**

$u_3 = A(-2)^3 + B(3)^3 + a + b.2^3 = 36, \qquad \Rightarrow -8A + 27B + a + 8b = 36$ **D**

$(2 \times \mathbf{A}) + \mathbf{B}$: $5B + 3a + 4b = 7$ **E**

$(2 \times \mathbf{B}) + \mathbf{C}$: $15B + 3a + 8b = 22$ **F**

$(2 \times \mathbf{C}) + \mathbf{D}$: $45B + 3a + 16b = 68$ **G**

$\mathbf{F} - \mathbf{E}$: $10B + 4b = 15$ **H**

$\mathbf{G} - \mathbf{F}$: $30B + 8b = 46, \qquad \Rightarrow 15B + 4b = 23$ **I**

$\mathbf{I} - \mathbf{H}$: $5B = 8, \qquad \Rightarrow B = 1.6$

$\Rightarrow$ in **H**: $4b = 15 - 10(1.6)$, $\Rightarrow b = -0.25$

$\Rightarrow$ in **E**: $3a = 7 - 5(1.6) - 4(-0.25)$, $\Rightarrow a = 0$

$\Rightarrow$ in **A**: $A + 1.6 - 0.25 = 2$, $\Rightarrow A = 0.65$

$\Rightarrow u_n = 0.65(-2)^n + 1.6(3)^n - 0.25(2^n)$.

(b) $u_{10} = 0.65(-2)^{10} + 1.6(3)^{10} - 0.25(2^{10}) = 94{,}888.$

Exercise 12F: Q14(e) should be a further 6 years, not 8 years.

Chapter 12 Answers
Exercise 12A (p.292): Q14 80 m

Exercise 12B (p.293): Q3(c) $T_{n+1} = T_n + 2n + 1,\ T_1 = 2$ or $T_{n+2} = 2T_{n+1} - T_n + 2,\ T_1 = 2,\ T_2 = 5$
Q5(c) –66
Q11(a) ¼, ⅗, ¼, ⅗, ¼, ⅗, (b) ⅗
Q13) …, 13, 25, 49, 94.

Exercise 12C (p.293): Q12(f) 9 weeks, (k) 2745 kg.

Exercise 12D (p.293): Q6(c) €316.
Q8(c) €211.05.

Exercise 12E (p.293): Q1(d) $x^2 - 7x + 10 = 0$
Q6(a) $T_n = \frac{19}{24}(3^n) + \frac{11}{40}(-5)^n$, (b) $-13{,}287{,}493$

Exercise 12F (p.294): Q15 (a) $M_n = 447.06\left(\frac{11}{10}\right)^n - 29.213\left(-\frac{10}{11}\right)^n - 67.846 - 10.476n$, (b) 976, not quite.

Chapter 13

Example 13C2 (p.227): final answer should be: $= \frac{5}{2} - \sqrt{3} = 0.770$

Example 13E2(b) (p.233): there are a number of sign errors in the solution

(b) $\int e^x \cos x\ dx$

Let $u = \cos x$, $\Rightarrow \dfrac{du}{dx} = -\sin x$, $\Rightarrow du = -\sin x\ dx$

and let $dv = e^x\ dx$, $\Rightarrow \dfrac{dv}{dx} = e^x$, $\Rightarrow v = e^x$

$\int u\ dv = uv - \int v\ du$

$\Rightarrow \int e^x \cos x\ dx = e^x \cos x + \int e^x \sin x\ dx$

To integrate $\int e^x \sin x\ dx$ we do another integration by parts.

Let $u = \sin x$, $\Rightarrow \dfrac{du}{dx} = \cos x$, $\Rightarrow du = \cos x\ dx$

and let $dv = e^x\ dx$, $\Rightarrow \dfrac{dv}{dx} = e^x$, $\Rightarrow v = e^x$

$\int u\ dv = uv - \int v\ du$

$\Rightarrow \int e^x \sin x\ dx = e^x \sin x - \int e^x \cos x\ dx$

$$\Rightarrow \int e^x \cos x\ dx = e^x \cos x + \left(e^x \sin x - \int e^x \cos x\ dx\right) = e^x \cos x + e^x \sin x - \int e^x \cos x\ dx$$

$$\Rightarrow 2\int e^x \cos x\ dx = e^x \cos x + e^x \sin x + C$$

$$\Rightarrow \int e^x \cos x\ dx = \frac{e^x}{2}\left(\cos x + \sin x\right) + D$$

Example 13F3(b) (p.236): small error in calculation

(b) In this example the quadratic expression in the denominator has a repeated root. To deal with this in partial fractions one of the fractions has this root normally, and the other has it squared.

$$\int \frac{2x+3}{x^2-6x+9}\ dx = \int\left(\frac{2}{x-3} + 9\left(x-3\right)^{-2}\right)\ dx$$

$$= 2\ln\left(x-3\right) + \frac{9\left(x-3\right)^{-1}}{-1} + C = 2\ln\left(x-3\right) - \frac{9}{\left(x-3\right)} + C$$

$$\frac{2x+3}{x^2-6x+9} = \frac{2x+3}{\left(x-3\right)^2} = \frac{A}{x-3} + \frac{B}{\left(x-3\right)^2}$$

$$\Rightarrow \frac{2x+3}{\left(x-3\right)^2} = \frac{A\left(x-3\right)+B}{\left(x-3\right)^2} = \frac{Ax-3A+B}{\left(x-3\right)^2}$$

$$\Rightarrow A = 2 \quad \text{and} \quad -3A+B=3, \qquad \Rightarrow B = 9$$

$$\Rightarrow \frac{2x+3}{x^2-6x+9} = \frac{2}{x-3} + \frac{9}{\left(x-3\right)^2}$$

Chapter 13 Answers

Exercise 13A (p.294): Q4(b) $\dfrac{3\sin x^2 - 2x(3x-4)\cos x^2 \ln\left(3x-4\right)}{\sin^2 x^2\left(3x-4\right)}$

Q4(c) $-4\sqrt{3x}\cos 2x + \dfrac{\sqrt{3}\cos^2 2x}{2\sqrt{x}}$

Q4(d) $-\dfrac{2e^{1-4x}}{\sqrt[3]{x^{10}}}\left(4\sqrt[3]{x^5} + \dfrac{5\sqrt[3]{x^2}}{3}\right)$

Q6 $-\dfrac{19}{16}$

Q9(b) $e^{\sin^2 x}\sin 2x$

Q9(c) $\dfrac{-2x}{\sqrt{\left(1-2x^2\right)^3}}\sin\left(\dfrac{1}{\sqrt{1-2x^2}}\right)$

Exercise 13C (p.294): Q1(e) $2x + 3\ln\left|\sec x\right| + C$

Exercise 13D (p.295): Q3(c) $\dfrac{\left(3e^x-4\right)^6}{9} + C$

Q4(b) $\dfrac{3}{2x} + \dfrac{\ln x}{2} + C$

Q4(c) $-2x - 8\ln\left(4-x\right) + D$

Q6(b) $\ln\left(e^x+6\right) + C$

Q6(f) $-\dfrac{\sqrt{e^{2-x^2}}}{3} + C$

Exercise 13E (p.295): Q2 $\frac{\cos x\left(\cos^2 x-3\right)}{3}+C$

Q4(a) $\frac{e^{2x}\left(2x^2-2x+1\right)}{4}+C$

Q4(c) $\frac{x^2\sin 4x}{4}+\frac{x\cos 4x}{8}-\frac{\sin 4x}{32}+C$

Q5(g) –0.793

Exercise 13F (p.295): Q5(a) $\frac{x^2}{2}+7x+17\ln\left(x-2\right)+C$

Q5(c) $x+\ln\left(x^2+1\right)-9\tan^{-1}x+C$

Q9(g) –0.901
Q11(b) 0.0512
Q11(c) 13120
Q11(d) 908.8
Q11(h) 17.0
Q11(k) 0.507

Chapter 14

Exercise 14A (p.242): Q13: should be 31.5 m s^{-1}, not 27 m s^{-1}.
Q14(a) & (b): should be minimum, not maximum.

Exercise 14B (p.247): Q9: $v=3+3t,$ not $3+4t$ for the first section of motion.
Q14(a) & (b): should be minimum, not maximum.

Example 14C1(a) (p.248): error in calculation in the solution

(a) $\vec{v}=\frac{1}{2}t^2\vec{i}-3t\vec{j},\ \Rightarrow$ the position vector $\vec{r}=\int\vec{v}\ dt=\int\left(\frac{1}{2}t^2\vec{i}-3t\vec{j}\right)\ dt=\frac{t^3}{6}\vec{i}-\frac{3t^2}{2}\vec{j}+\vec{C}$

When $t=0,\ \vec{r}=2\vec{i}+18\vec{j}$ m, $\quad\Rightarrow\vec{r}=\frac{0^3}{6}\vec{i}-\frac{3(0)^2}{2}\vec{j}+\vec{C}=2\vec{i}+18\vec{j},\quad\Rightarrow\vec{C}=2\vec{i}+18\vec{j}$

$$\Rightarrow\vec{r}=\frac{t^3}{6}\vec{i}-\frac{3t^2}{2}\vec{j}+2\vec{i}+18\vec{j}=\left(2+\frac{t^3}{6}\right)\vec{i}+\left(18-\frac{3t^2}{2}\right)\vec{j}\text{ m}$$

When $t=4,\ \vec{r}=\left(2+\frac{4^3}{6}\right)\vec{i}+\left(18-\frac{3(4)^2}{2}\right)\vec{j}=\frac{38}{3}\vec{i}-6\vec{j}$ m

$$\Rightarrow\left|\vec{r}\right|=\sqrt{\left(\frac{38}{3}\right)^2+\left(-6\right)^2}=14.0\text{ m and }\tan\alpha=\frac{6}{\frac{38}{3}}=\frac{18}{38}=\frac{9}{19},\ \Rightarrow\alpha=25.3^\circ\text{ below horizontal.}$$

Exercise 14C (p.252): Q12(a) & (b): should be maximum velocity, not acceleration.
Q15(b): should be $\left(120\vec{i}+80\vec{j}\right)$ m s^{-1}, not $\left(160\vec{i}+160\vec{j}\right)$ m.

Extra question – Q16
A particle is moving in a horizontal circle of centre O, with radius r and constant angular speed ω.
(a) Show that the displacement of the particle relative to O at any time t is $\vec{s}=r\cos\omega t\vec{i}+r\sin\omega t\vec{j}$ m. Note that at time t = 0, $\vec{s}$ is along the $\vec{i}$ axis.
(b) Derive an expression for $\vec{v}$, the velocity of the particle at any time t.
(c) Use a dot product calculation to show that the particle's velocity and displacement are always perpendicular to each other.
(d) Show that the acceleration of the particle is always directed towards the centre O.

Chapter 14 Answers

Exercise 14A (p.296): Q4(d) 7 m s^{-2}
Q5(b) 7 m s^{-1}
Q12(c) 41.3 m s^{-1}
Q12(e) 114 m
Q14(b) –0.135 m s^{-1}
Q16(d) 24.1
Q19(b) 33.1 m
Q21(b) 71.0 m.

Exercise 14B (p.296): Q5(d) 35.1 m
Q9(d) 20.75 m
Q13(b) 0 s, 8 m s^{-2}
Q14(g) 11.3 m

Exercise 14C (p.296): Q9(a) 22.3 m, 17.2° N of W
Q9(c) 175.4°
Q12(a) 5.83 m s^{-2}
Q12(c) 3.44 m
Q13(a) $\left(\frac{2}{3}t^3 - 2t + 6\right)\vec{i} + \left(2 - 2.5t^2\right)\vec{j}$ m,
Q13(b) 7 m
Q15(c) 301.3 m s^{-1}.
Q16(b) $\vec{v} = -r\omega \sin \omega t \vec{i} + r\omega \cos \omega t \vec{j}$ m s^{-1}.

Chapter 15 Answers

Exercise 15A (p.296): Q8(j) $y = \frac{4^{12} e^{\frac{x}{2}(x+4)}}{(x+4)^{12}}$

Q8(m) $y = x + 3\ln\left(\frac{x^2+4}{4}\right) - 4\tan^{-1}\frac{x}{2} + 2$

Exercise 15F (p.297): answers to questions 4, 5 and 6 are numbered in the wrong order.