Chapter 5
Collisions and Impacts

Section 5A: Balls Bouncing on Horizontal Floors

When a particle hits off a solid surface, it bounces back. For example if you drop a tennis ball onto a hard floor, it bounces back up to a certain height. Different types of particles and different surfaces have different amounts of bounciness, and this determines how high something will bounce. This was all explained by Newton, in **Newton's Experimental Law of Restitution**. Restitution is just another word for bounciness. There are two versions of Newton's Law. We will use a simple one in this section for collision between one moving object and one fixed object. In the next section we will see another version of the law for collision between two moving objects.

Newton's Experimental Law of Restitution involves a concept called the **Coefficient of Restitution,** for which we use the symbol ***e***. This is a property of the particles and surfaces involved, and is basically a measure of how bouncy or elastic something is. The values of the coefficient of restitution (e) go from 0 to 1, i.e. $0 \le e \le 1$. If you drop a bouncy ball from a height, it bounces to almost the same height again. This would have a coefficient of restitution (e) of almost 1. If e = 1 this is called perfectly elastic. If you drop a block of stone from the same height, it doesn't really bounce at all. This is because the coefficient of restitution (e) is about zero. If e = 0 this is called perfectly inelastic.

The formula we use for the coefficient of restitution (*e*) is: $$-e = \frac{\text{new velocity}}{\text{old velocity}} = \frac{v}{u}$$

Solution Strategy for problems involving particles bouncing off the ground:

1) Find the velocity with which the particle hits the ground using $v^2 = u^2 + 2as$.
2) To calculate the velocity after the bounce use $-e = \frac{v}{u}$.
3) Find the height to which the particle then rises again using $v^2 = u^2 + 2as$.

Example 5A1

A ball falls vertically from a height of 2 m onto a horizontal floor. The coefficient of restitution between the ball and the floor is 0.9.

(i) Find the speed with which the ball hits the floor.
(ii) Find the speed of the ball just after the first bounce.
(iii) Find the height to which the ball then rises.

Solution:

(i) $u = 0,\ a = 10 \text{ m s}^{-2},\ s = 2 \text{ m},\ v = ?$ $\qquad v^2 = u^2 + 2as$

$\Rightarrow v^2 = 0 + 2(10)(2)$ $\qquad \Rightarrow v = \sqrt{40} = 6.32 \text{ m s}^{-1}$

(ii) This now becomes the u for the next part, and we must calculate the v after the bounce, using: $-e = \frac{v}{u}$. If we make up positive, then u is negative, i.e. $u = -6.32 \text{ m s}^{-1}$

$\Rightarrow -0.9 = \frac{v}{-6.32}$ $\qquad \Rightarrow v = 0.9(6.32) = 5.69 \text{ m s}^{-1}$

(iii) This is now the u for the ball rising vertically to reach a new maximum height.

$u = 5.69 \text{ m s}^{-1},\ a = -10 \text{ m s}^{-2},\ v = 0,\ s = h$ $\qquad v^2 = u^2 + 2as$

$\Rightarrow 0 = (5.69)^2 - 20h$ $\qquad \Rightarrow h = \frac{32.4}{20} = 1.62 \text{ m.}$

Exercise 5A – balls bouncing on the ground

1) A ball is dropped from rest from a height of 1.8 m onto a smooth horizontal table. The ball hits the table with a speed of v m s^{-1} and then rebounds to a height of h metres above the table.
The coefficient of restitution between the ball and the table is ⅔.
   (i) Find the value of v.
   (ii) Find the value of h.

2) A ball is dropped from rest from a height of 5 m onto a smooth horizontal floor. The ball hits the floor and rebounds to a height of h metres above the floor.
The coefficient of restitution between the ball and the floor is $\frac{4}{5}$.
   (i) Find the speed of the ball when it hits the floor.
   (ii) Find the value of h.

3) A ball is dropped from rest from a height of 3.2 m onto a smooth horizontal floor. The ball hits the floor and rebounds to a height of 1.8 m above the floor.
The coefficient of restitution between the ball and the floor is e.
   (i) Find the speed of the ball when it hits the floor.
   (ii) Find the value of e.

4) A ball is fired vertically down with a speed of 4 m s^{-1} from a height of 6.4 m above a smooth horizontal floor. The ball hits the floor and rebounds to a height of 3.2 metres above the floor.
The coefficient of restitution between the ball and the floor is e.
   (i) Find the speed of the ball when it hits the floor.
   (ii) Find the value of e.

Section 5B: 2 Particle Direct Collisions

Momentum

The momentum of a moving body is simply its mass multiplied by its velocity. It is important to note that momentum is a vector quantity. The normal unit of momentum is the Newton second (Ns) or the kilogram metre per second (kg m s^{-1}). Note that 1 kg m s^{-1} = 1 Ns.

Momentum = Mass x Velocity

You can think of momentum as a measure of how difficult it is to stop a moving body. For example, a large body with a low velocity, such as a moving truck, could have the same momentum as a small body with a large velocity, such as a bullet. These two bodies, if they have the same momentum, would be equally difficult to stop, even though they have vastly different masses.

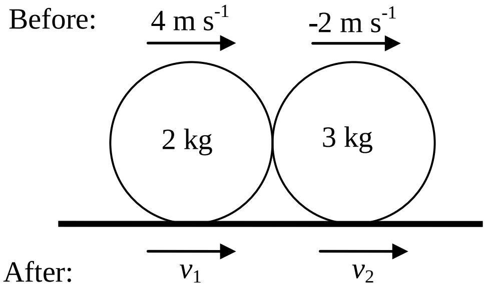
The Principle of Conservation of Momentum

In any interaction between two or more bodies, the total momentum of the system of bodies before the interaction is equal to the total momentum of the system of bodies after the interaction, provided no external force acts on the bodies.

In general we use this when two bodies collide into each other. We write this as a formula, when two bodies collide into each other and bounce off one another, as:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

In this section we will look at problems where two moving particles collide with each other in a straight line. We call these direct collisions. The first step in any problem is to draw a simple diagram. We represent the particles as spheres, and write the mass inside each sphere. Write the velocities before the collision above each sphere. We generally make to the right as the positive direction. If a particle is moving left, make sure to write its velocity as negative. Then write in the unknown velocities after the collision below the diagram, generally calling them v_1 and v_2



Two laws are then used. The first of these is the **Principle of Conservation of Momentum** (PCM), as described above. We use this as the formula: $\boldsymbol{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$. Applying this to the example above:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$\Rightarrow 2(4) + 3(-2) = 2v_1 + 3v_2$$
$$\Rightarrow 2v_1 + 3v_2 = 8 - 6 = 2 \qquad \textbf{A}$$

Next **Newton's Experimental Law of Restitution** (NELR) is used. Previously we saw this used in connection with a moving particle hitting a fixed surface. In this case there are two moving particles, so the version of NELR we used is different. It still involved the coefficient of restitution the same as before. The formula we use for direct collisions is:

$$\boxed{-e = \frac{v_2 - v_1}{u_2 - u_1}}$$

[N.B. This can also be written as $-e = \frac{v_1 - v_2}{u_1 - u_2}$*, or as* $v_2 - v_1 = -e(u_2 - u_1)$ *which are equivalent. The version to the left is preferred as it makes the calculations a bit more straightforward.]*

Applying this to the example above, taking the value of the coefficient of restitution e as 0.5, gives us a second equation as follows:

$$-e = \frac{v_2 - v_1}{u_2 - u_1}, \qquad \Rightarrow -0.5 = \frac{v_2 - v_1}{-2 - 4}$$
$$\Rightarrow v_2 - v_1 = 3 \qquad \textbf{B}$$

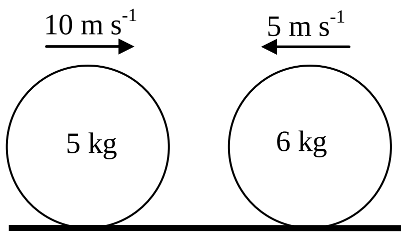
The two equations obtained from the Principle of Conservation of Momentum and Newton's Experimental Law of Restitution are then solved using simultaneous equations. If the initial velocities and coefficient of restitution are given numerically in the question, the actual values of v_1 and v_2 can be calculated. If however the initial velocities and / or the coefficient of restitution are given algebraically, then v_1 and v_2 will be algebraic expressions also.

Solution Strategy for problems with direct collisions:

1) Draw a **simple diagram** showing any known velocities. Put all velocities to the right, and if they are actually to the left, mark them as negative.
2) Use the **Principle of Conservation of Momentum** to get one equation: $\boldsymbol{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$.
3) Use **Newton's Experimental Law of Restitution** to get a second equation: $\boldsymbol{-e = \frac{v_2 - v_1}{u_2 - u_1}}$.
4) Solve these two equations using **simultaneous equations to find v_1 and v_2.**

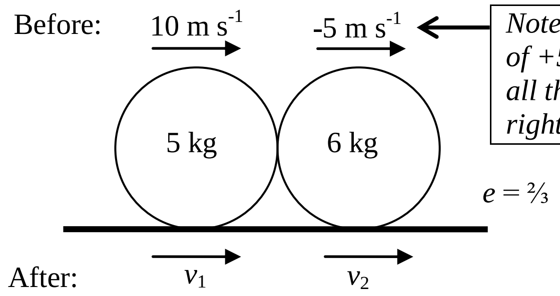
Example 5B1

A smooth sphere, of mass 5 kg, moving with constant velocity 10 m s^{-1}, collides with a second smooth sphere, of mass 6 kg, moving in the same straight line, but in the opposite direction with a speed of 5 m s^{-1}.
The coefficient of restitution for the collision is ⅔.



Find the speed of each of the two spheres after the collision.

Solution:



Note that we write this as -5 m s^{-1} to the right instead of +5 m s^{-1} to the left. This is because it must go into all the formulae as -5, as we take the direction to the right as positive.

$e = \frac{2}{3}$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$\Rightarrow 5(10) + 6(-5) = 5v_1 + 6v_2$$
$$\Rightarrow 20 = 5v_1 + 6v_2 \quad \textbf{A}$$

$$-e = \frac{v_2 - v_1}{u_2 - u_1}, \qquad \Rightarrow -\frac{2}{3} = \frac{v_2 - v_1}{-5-10}$$
$$\Rightarrow -\frac{2}{3}(-15) = 10 = v_2 - v_1 \quad \textbf{B}$$

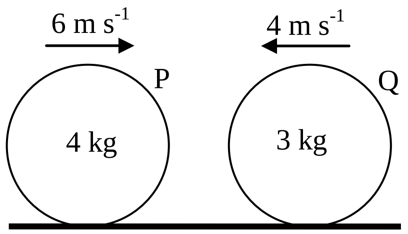
$$\left.\begin{array}{l} 1\times\textbf{A}: \ 20 = 5v_1 + 6v_2 \\ 5\times\textbf{B}: \ 50 = 5v_2 - 5v_1 \end{array}\right\} +$$

$$\Rightarrow 70 = 11v_2, \qquad \Rightarrow v_2 = \frac{70}{11} \text{ m s}^{-1}$$

In **B**: $v_1 = v_2 - 10 = \frac{70}{11} - 10 = -\frac{40}{11}$ m s^{-1}

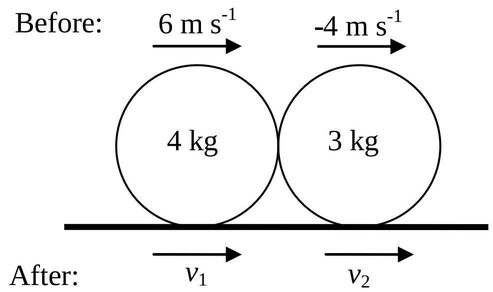
Example 5B2

Two smooth spheres P and Q, of masses 4 kg and 3 kg respectively, and travelling in opposite directions with speeds 6 m s^{-1} and 4 m s^{-1} respectively, collide directly on a smooth horizontal table. The coefficient of restitution for the collision is e. As a result of the collision the change in the linear momentum of Q is 40e N s.



(i) Show that the speed of P after the collision is $6 - 10e$.
(ii) Find the value of e.
(iii) Find the speed of Q after the collision.

Solution:



(i) The change in momentum on Q $= m_2v_2 - m_2u_2$

$$\Rightarrow 40e = 3v_2 - 3(-4)$$
$$\Rightarrow 40e = 3v_2 + 12$$
$$\Rightarrow v_2 = \frac{40e - 12}{3}$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$\Rightarrow 4(6) + 3(-4) = 4v_1 + 3\left(\frac{40e-12}{3}\right)$$
$$\Rightarrow 12 = 4v_1 + 40e - 12$$
$$\Rightarrow v_1 = \frac{24 - 40e}{4} = 6 - 10e \quad \text{Q.E.D.}$$

(ii) $-e = \frac{v_2 - v_1}{u_2 - u_1}, \qquad \Rightarrow -e = \frac{\frac{40e-12}{3} - (6 - 10e)}{-4-6}$

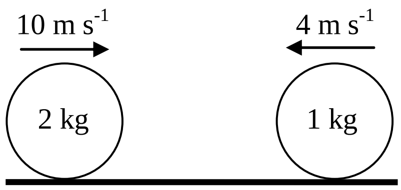
$$\Rightarrow 10e = \frac{40e-12}{3} - 6 + 10e, \qquad \Rightarrow 30e = 40e - 12 - 18 + 30e$$

$$\Rightarrow 30 = 40e, \qquad \Rightarrow e = \frac{3}{4}$$

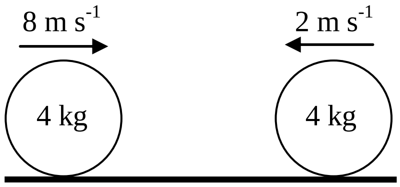
(iii) $v_2 = \frac{40e - 12}{3} = v_2 = \frac{40\left(\frac{3}{4}\right) - 12}{3} = \frac{18}{3} = 6 \text{ m s}^{-1}$

Exercise 5B – 2 particle direct collisions

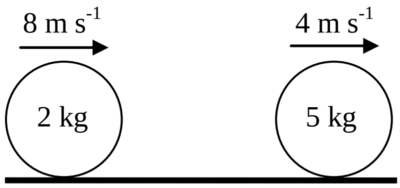
1) A smooth sphere, of mass 2 kg, moving with constant velocity 10 m s^{-1}, collides with a second smooth sphere, of mass 1 kg, moving in the same straight line, but in the opposite direction with a speed of 4 m s^{-1}.
The coefficient of restitution for the collision is ½.
Find the speed of each of the two spheres after the collision.



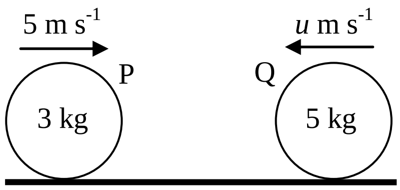
2) A smooth sphere, of mass 4 kg, moving with constant velocity 8 m s^{-1}, collides with a second smooth sphere, also of mass 4 kg, moving in the same straight line, but in the opposite direction with a speed of 2 m s^{-1}.
The coefficient of restitution for the collision is 0.8.
Find the speed of each of the two spheres after the collision.



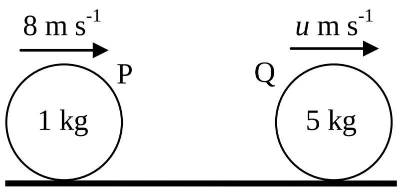
3) A smooth sphere, of mass 2 kg, moving with constant velocity 8 m s^{-1}, collides with a second smooth sphere, of mass 5 kg, moving in the same straight line, and in the same direction with a speed of 4 m s^{-1}.
The coefficient of restitution for the collision is ¾.
Find the speed of each of the two spheres after the collision.



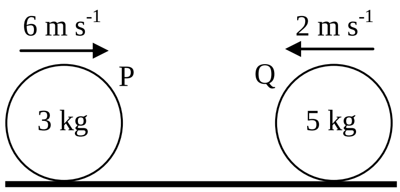
4) A smooth sphere P, of mass 3 kg, moving with a speed of 5 m s^{-1}, collides directly with a smooth sphere Q, of mass 5 kg, moving in the opposite direction with a speed of u m s^{-1} on a smooth horizontal table.
The coefficient of restitution for the collision is ⅓.
As a result of the collision sphere P is brought to rest.
   (i) Find the value of u.
   (ii) Find the speed of Q after the collision.



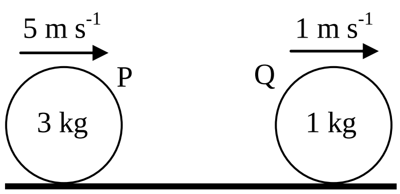
5) A smooth sphere P, of mass 1 kg, moving with a speed of 8 m s^{-1}, collides directly with a smooth sphere Q, of mass 5 kg, moving in the same direction with a speed of u m s^{-1} on a smooth horizontal table.
The coefficient of restitution for the collision is 0.6.
As a result of the collision sphere P is brought to rest.
   (i) Find the value of u.
   (ii) Find the speed of Q after the collision.



6) Two smooth spheres P and Q, of masses 3 kg and 5 kg respectively, and travelling in opposite directions with speeds 6 m s^{-1} and 2 m s^{-1} respectively, collide directly on a smooth horizontal table. The coefficient of restitution for the collision is e. As a result of the collision the change in the linear momentum of Q is 45e N s.
   (i) Show that the speed of P after the collision is $6 - 15e$.
   (ii) Find the value of e.
   (iii) Find the speed of Q after the collision.



7) Two smooth spheres P and Q, of masses 3 kg and 1 kg respectively, and travelling in the same direction with speeds 5 m s^{-1} and 1 m s^{-1} respectively, collide directly on a smooth horizontal table. The coefficient of restitution for the collision is e. As a result of the collision the change in the linear momentum of Q is 7e N s.
   (i) Show that the speed of P after the collision is $5 - \frac{7}{3}e$.
   (ii) Find the value of e.
   (iii) Find the speed of Q after the collision.



Section 5C: 2 Particle Direct Collisions with Energy and Impulse

Impulse: From Newton's Second Law of Motion, we know that: $\vec{F} = m\vec{a}$.

We can then write this as: $\vec{F} = \dfrac{m\left(\vec{v} - \vec{u}\right)}{t} = \dfrac{m\vec{v} - m\vec{u}}{t}$

$\Rightarrow \vec{F}t = m\vec{v} - m\vec{u}$

but $\vec{F}t =$ resultant force $\times$ time

and $m\vec{v} - m\vec{u} =$ change in momentum

What this means is that the resultant force multiplied by the time of contact of the resultant force equals the change in momentum. If we take the example of a golfer hitting a golf ball, it is very difficult to calculate the force with which the golf club hits the ball, and it is also extremely difficult to work out the time for which the club and the ball are in contact. However if we know the velocity with which the ball leaves, we can calculate the change in momentum, and this change in momentum will be equal to the product of the resultant force by the time of contact. We call this concept impulse.

Impulse is used to measure the overall effect of a force. The size of a force does not tell us directly the overall effect of that force, as the time for which the force acts is equally important. For example a small force acting for a long time could have the same effect as a large force acting for a short time.

We calculate the impulse by finding the change in momentum of the body. However it is worth remembering that the impulse is also equal to the product of the force and the time of contact. Impulse is a vector quantity. The unit of impulse is the same as the unit of momentum, i.e. the kilogram metre per second (kg m s^{-1}) or the Newton second (Ns). We use the symbol $\vec{I}$ for impulse.

Impulse = Change in Momentum

$$\boldsymbol{\vec{I} = m\vec{v} - m\vec{u}}$$

Kinetic Energy: The kinetic energy (K.E.) of a body is the energy a body has because it is moving.

Kinetic Energy is calculated using the formula: kinetic energy = ½ $m\,v^2$, where m is the mass of the body (in kg) and v is its speed (in m s^{-1}).

Kinetic Energy = ½ $\boldsymbol{m\,v^2}$

Since, by definition, energy is the capacity of a body to do work, the unit of energy is the same as that of work, i.e. the joule (J), with 1 joule being equal to 1 newton metre.

Loss of Kinetic Energy: When a collision happens, overall some of the combined kinetic energy of the particles in the collision is lost. While, according to the Principle of Conservation of Energy, energy cannot be created or destroyed, what happens here is that some of the kinetic energy that the particles had before the collision is converted during the collision to other forms of energy, such as sound energy and heat energy, and so after the collision the total kinetic energy of the particles is less than it was before the collision.

To calculate the loss of kinetic energy in a collision, find the total kinetic energy of the two particles before the collision, and subtract the total kinetic energy of the two particles after the collision. Note that often one of the individual particles can actually increase its kinetic energy during the collision (i.e. it is moving faster after the collision than before) but the overall kinetic energy of the whole system never increases.

Loss of Kinetic Energy = Total Kinetic Energy before Collision - Total Kinetic Energy after Collision

$$\textbf{Loss of K.E.} = \left(\tfrac{1}{2}\boldsymbol{m_1 u_1}^2 + \tfrac{1}{2}\boldsymbol{m_2 u_2}^2\right) - \left(\tfrac{1}{2}\boldsymbol{m_1 v_1}^2 + \tfrac{1}{2}\boldsymbol{m_2 v_2}^2\right)$$

$$\textbf{Fraction of K.E. lost} = \frac{\textbf{Total K.E. lost}}{\textbf{Total K.E. before collision}}$$

Solution Strategy for problems with direct collisions with impulse and loss of kinetic energy:

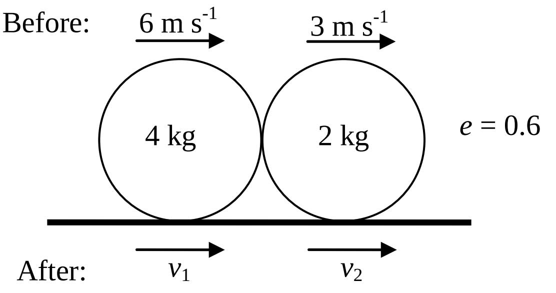
1) Draw a **simple diagram** showing any known velocities. Put all velocities to the right, and if they are actually to the left, mark them as negative.
2) Use the **Principle of Conservation of Momentum** to get one equation: $\boldsymbol{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$.
3) Use **Newton's Experimental Law of Restitution** to get a second equation: $\boldsymbol{-e = \frac{v_2 - v_1}{u_2 - u_1}}$.
4) Solve these two equations using **simultaneous equations to find $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$.**

5A) If the **impulse** is required, use $\boldsymbol{\vec{I} = m\vec{v} - m\vec{u}}$ for both particles.
5B) If the **loss of kinetic energy** is required, find the kinetic energy of both particles both before and after the collision, and then subtract the total K.E. after from the total K.E. before, using $\boldsymbol{K.E. = \frac{1}{2}mv^2}$.

Example 5C1
A smooth sphere A, of mass 4 kg, moving with speed 6 m s^{-1} collides directly with a smooth sphere B, of mass 2 kg, moving in the same direction with speed 3 m s^{-1}. The coefficient of restitution between the spheres is 0.6. Find:

(i) the speed of each sphere after the collision,
(ii) the impulse imparted to each sphere during the collision,
(iii) the loss of kinetic energy during the collision.

Solution:



(i) $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow 4(6) + 2(3) = 4v_1 + 2v_2$

$\Rightarrow 4v_1 + 2v_2 = 30$

$\Rightarrow 2v_1 + v_2 = 15$ **A**

$-e = \frac{v_2 - v_1}{u_2 - u_1}$

$\Rightarrow -0.6 = \frac{v_2 - v_1}{3 - 6}$, $\Rightarrow v_2 - v_1 = -0.6(-3) = 1.8$ **B**

$1 \times \mathbf{A}: \quad 2v_1 + v_2 = 15$
$2 \times \mathbf{B}: \quad 2v_2 - 2v_1 = 3.6$ } +

$\Rightarrow 3v_2 = 18.6$ $\Rightarrow v_2 = 6.2$ m s^{-1}

In **B**: $v_1 = v_2 - 1.8 = 6.2 - 1.8 = 4.4$ m s^{-1}.

(ii) Impulse: $\vec{I} = m\vec{v} - m\vec{u}$

Impulse on A: $I_A = 4(4.4) - 4(6) = -6.4$ N s.

Impulse on B: $I_B = 2(6.2) - 2(3) = 6.4$ N s.

Note if you only have to find the magnitude of the impulse, use the right hand sphere, as the impulse on it is positive. As expected from Newton's 3rd Law of Motion, these are equal and opposite.

(iii) $\text{K.E.} = \frac{1}{2}mv^2$

K.E. before $= \frac{1}{2}(4)(6)^2 + \frac{1}{2}(2)(3)^2 = 81$ J.

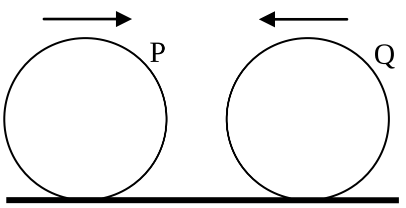
K.E. after $= \frac{1}{2}(4)(4.4)^2 + \frac{1}{2}(2)(6.2)^2 = 77.16$ J.

Loss of K.E. = K.E. before – K.E. after

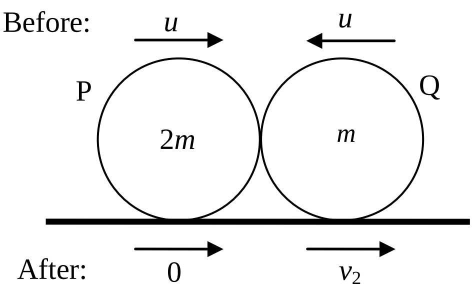
$\Rightarrow$ Loss of K.E. $= 81 - 77.16 = 3.84$ J.

Example 5C2
Two smooth spheres P and Q collide directly on a smooth horizontal table. The mass of P is twice the mass of Q. Before collision, P and Q are moving in opposite directions with equal speeds. As a result of the collision sphere P comes to rest.

(i) Find the coefficient of restitution for the collision.
(ii) Find also, the fraction of the total kinetic energy lost as a result of the collision.



Solution: (i) Since we are not given specific value for mass and velocity in the question, we call both velocities u and the mass of P is $2m$ and the mass of Q is m.



$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$\Rightarrow 2m(u) + m(-u) = 2m(0) + 2mv_2$$
$$\Rightarrow mu = 2mv_2$$
$$\Rightarrow v_2 = \frac{u}{2}$$

$$-e = \frac{v_2 - v_1}{u_2 - u_1}, \quad \Rightarrow -e = \frac{\frac{u}{2} - 0}{-u - u}, \quad \Rightarrow -e = \frac{-1}{4}, \quad \Rightarrow e = \frac{1}{4}$$

(iii) $\text{K.E.} = \tfrac{1}{2}mv^2$

$$\text{K.E. before} = \tfrac{1}{2}(2m)(u)^2 + \tfrac{1}{2}(m)(u)^2 = \tfrac{3}{2}mu^2$$

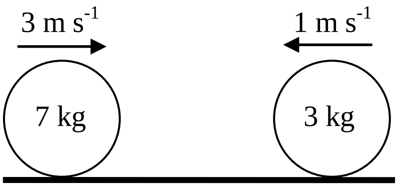
$$\text{K.E. after} = \tfrac{1}{2}(2m)(0)^2 + \tfrac{1}{2}(m)\left(\frac{u}{2}\right)^2 = \tfrac{1}{8}mu^2$$

$$\text{Loss of K.E.} = \text{K.E. before} - \text{K.E. after} = \tfrac{3}{2}mu^2 - \tfrac{1}{8}mu^2 = \tfrac{11}{8}mu^2$$

$$\text{Fraction of K.E. lost} = \frac{\text{Loss of K.E.}}{\text{K.E. before}} = \frac{\tfrac{11}{8}mu^2}{\tfrac{3}{2}mu^2} = \frac{11}{8} \times \frac{2}{3} = \frac{11}{12}$$

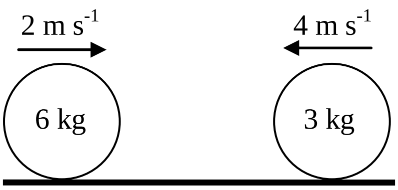
Exercise 5C – direct collision problems with impulse and loss of kinetic energy

1) A smooth sphere, of mass 7 kg, collides directly with another smooth sphere, of mass 3 kg, on a smooth horizontal table. The two spheres are moving in opposite directions with speeds of 3 m s^{-1} and 1 m s^{-1} respectively. The coefficient of restitution for the collision is ¾.



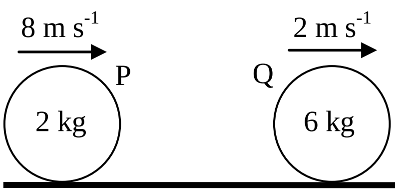
Find: (i) the speed of each of the spheres after the collision,
(ii) the loss in kinetic energy due to the collision,
(iii) the magnitude of the impulse imparted to both spheres in the collision.

2) A smooth sphere, of mass 6 kg, collides directly with another smooth sphere, of mass 3 kg, on a smooth horizontal table. The two spheres are moving in opposite directions with speeds of 2 m s^{-1} and 4 m s^{-1} respectively. The coefficient of restitution for the collision is ½.



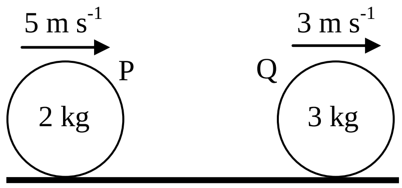
Find: (i) the speed of each of the spheres after the collision,
(ii) the fraction of kinetic energy lost in the collision,
(iii) the magnitude of the impulse imparted to both spheres in the collision.

3) A smooth sphere P, of mass 2 kg, collides directly with another smooth sphere Q, of mass 6 kg, on a smooth horizontal table. The two spheres are moving in the same direction with speeds of 8 m s^{-1} and 2 m s^{-1} respectively. The coefficient of restitution for the collision is $\tfrac{5}{6}$.



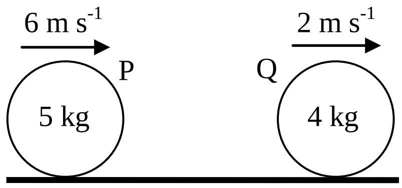
Find: (i) the speed of each of the spheres after the collision,
(ii) the loss in kinetic energy due to the collision,
(iii) the magnitude of the impulse imparted to sphere P in the collision.

4) A smooth sphere P, of mass 2 kg, collides directly with another smooth sphere Q, of mass 3 kg, on a smooth horizontal table. The two spheres are moving in the same direction with speeds of 5 m s^{-1} and 3 m s^{-1} respectively. The coefficient of restitution for the collision is ¼.



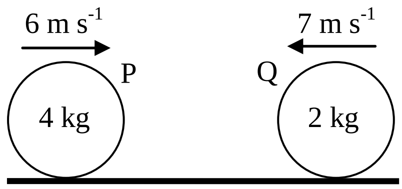
Find: (i) the speed of each of the spheres after the collision,
(ii) the change in the kinetic energy of P due to the collision,
(iii) the magnitude of the impulse imparted to sphere P in the collision.

5) A smooth sphere P, of mass 5 kg, collides directly with another smooth sphere Q, of mass 4 kg, on a smooth horizontal table. The two spheres are moving in the same direction with speeds of 6 m s^{-1} and 2 m s^{-1} respectively.
The impulse imparted to Q due to the collision is 12 N s.



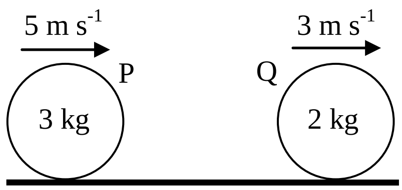
Find: (i) the speed of each of the spheres after the collision,
(ii) the coefficient of restitution for the collision,
(iii) the loss in kinetic energy due to the collision.

6) A smooth sphere P, of mass 4 kg, collides directly with another smooth sphere Q, of mass 2 kg, on a smooth horizontal table. The two spheres are moving in opposite directions with speeds of 6 m s^{-1} and 7 m s^{-1} respectively.
The coefficient of restitution for the collision is e. As a result of the collision P continues to move in the same direction with a speed of $e \text{ m s}^{-1}$.



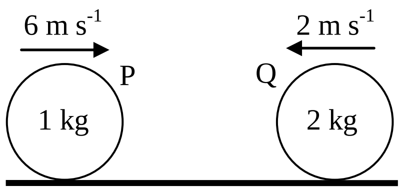
Find: (i) the value of e,
(ii) the loss in kinetic energy due to the collision, correct to one place of decimals.

7) A smooth sphere P, of mass 3 kg, collides directly with another smooth sphere Q, of mass 2 kg, on a smooth horizontal table. The two spheres are moving in the same direction with speeds of 5 m s^{-1} and 3 m s^{-1} respectively.
The coefficient of restitution for the collision is e.
After the collision, sphere Q continues to travel in the same direction but with a speed of 4.5 m s^{-1}.



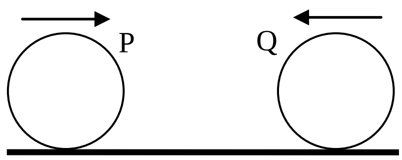
Find: (i) the speed of P after the collision,
(ii) the value of e,
(iii) the loss in kinetic energy due to the collision,
(iv) the magnitude of the impulse imparted to each sphere in the collision.

8) A smooth sphere P, of mass 1 kg, collides directly with another smooth sphere Q, of mass 2 kg, on a smooth horizontal table. The two spheres are moving in opposite directions with speeds of 6 m s^{-1} and 2 m s^{-1} respectively.
The coefficient of restitution for the collision is e.
As a result of the collision, sphere P is brought to rest.



Find: (i) the speed of Q after the collision,
(ii) the value of e,
(iii) the fraction of kinetic energy lost due to the collision.

9) Two smooth spheres P and Q collide directly on a smooth horizontal table. The mass of P is four times the mass of Q. Before collision, P and Q are moving in opposite directions with the speed of Q being twice that of P. As a result of the collision sphere P comes to rest.



Find: (i) the coefficient of restitution for the collision,
(ii) the fraction of kinetic energy lost due to the collision.

Section 5D: 3 Particle Direct Collisions

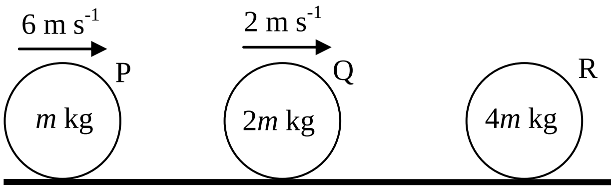
Collisions with three particles are much the same as collisions with 2 particles, except there will be a sequence of two or three different collisions, rather than just one. The method for dealing with each collision is exactly the same as before, using the Principle of Conservation of Momentum and Newton's Experimental Law of Restitution to get two different equations, and then solving these using simultaneous equations.

Solution Strategy for problems with direct collisions with 3 particles:

1) Draw a **simple diagram** showing any known velocities. Put all velocities to the right, and if they are actually to the left, mark them as negative.
2) Use the **Principle of Conservation of Momentum** to get one equation: $\boldsymbol{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$.
3) Use **Newton's Experimental Law of Restitution** to get a second equation: $\boldsymbol{-e = \dfrac{v_2 - v_1}{u_2 - u_1}}$.
4) Solve these two equations using **simultaneous equations to find v_1 and v_2.**
5) **Repeat the above steps** for any subsequent collisions.

Example 5D1

A smooth sphere P, of mass m kg, moving with a speed of 6 m s^{-1} collides directly with a second smooth sphere Q, of mass $2m$ kg, moving in the same direction with a speed of 2 m s^{-1} on a smooth horizontal table.



After the collision, spheres P and Q keep moving in the same direction and the ratio of their speeds is

velocity of P : velocity of Q = 2 : 3

(i) Find the speeds of spheres P and Q after this first collision has taken place.
(ii) Find the coefficient of restitution for this collision.

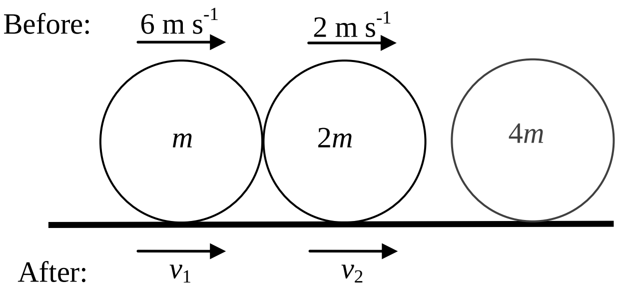
Sphere Q now goes on to collide directly with a stationary sphere R of mass $4m$ kg. This collision causes Q to lose 96% of its kinetic energy, but it continues in the same direction.

(iii) Find the speeds of spheres Q and R after this second collision has taken place.
(iv) Find the coefficient of restitution for this collision.

P then strikes Q again. Show that Q will go on to collide with R again.

Solution:

1st collision between P and Q



(i) $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow m(6) + 2m(2) = mv_1 + 2mv_2$

$\Rightarrow 10 = v_1 + 2v_2$ **A**

$\dfrac{\text{Velocity of P}}{\text{Velocity of Q}} = \dfrac{v_1}{v_2} = \dfrac{2}{3}$, $\Rightarrow 3v_1 = 2v_2$ **B**

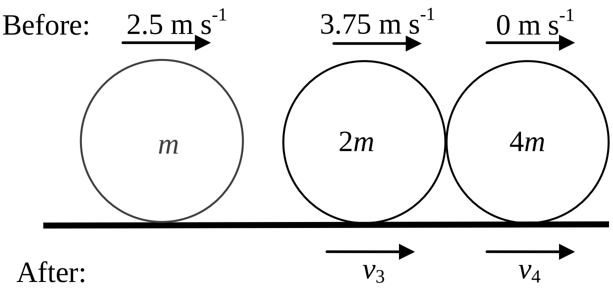
From **B** in **A**: $10 = v_1 + 3v_1$, $\Rightarrow v_1 = 2.5$ m s^{-1}

In **B**: $3(2.5) = 2v_2$, $\Rightarrow v_2 = 3.75$ m s^{-1}

(ii) $-e = \dfrac{v_2 - v_1}{u_2 - u_1}$, $\Rightarrow -e = \dfrac{3.75 - 2.5}{2 - 6} = -\dfrac{5}{16}$,

$\Rightarrow e = \dfrac{5}{16}$

2nd collision between Q and R



For a third collision, between P and Q

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow m(2.5) + 2m(0.75) = mv_5 + 2mv_6,$

$\Rightarrow 4 = v_5 + 2v_6$ **C**

$-e = \dfrac{v_2 - v_1}{u_2 - u_1}, \quad \Rightarrow -\dfrac{5}{16} = \dfrac{v_6 - v_5}{0.75 - 2.5},$

$\Rightarrow \dfrac{35}{64} = v_6 - v_5$ **D**

C + **D**: $3v_6 = 4.546875, \quad \Rightarrow v_6 = 1.515625 \text{ m s}^{-1}$

$\Rightarrow$ in **D**: $v_5 = v_6 - \dfrac{35}{64} = 0.96875 \text{ m s}^{-1}$

(iii) K.E. of Q before collision $= \frac{1}{2}(2m)(3.75)^2$
$= 14.0625m$ J

If 96% of the K.E. is lost, that leaves 4%

$= 0.04(14.0625m) = 0.5625m$ J

$\Rightarrow 0.5625m = \frac{1}{2}(2m)(v_3)^2, \quad \Rightarrow v_3 = 0.75 \text{ m s}^{-1}$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow 2m(3.75) + 4m(0) = 2m(0.75) + 4mv_4,$

$\Rightarrow v_4 = 1.5 \text{ m s}^{-1}$

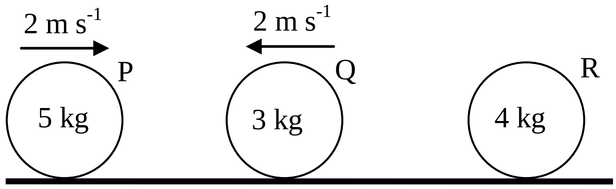
(iv) $-e = \dfrac{v_2 - v_1}{u_2 - u_1}, \quad \Rightarrow -e = \dfrac{1.5 - 0.75}{0 - 3.75} = -\dfrac{1}{5},$

$\Rightarrow e = \dfrac{1}{5}$

As Q is now moving with speed $1.515625 \text{ m s}^{-1}$ and R is moving with speed 1.5 m s^{-1}, this means that Q will catch up with R again and there will be another collision.

Exercise 5D – direct collision problems with inequalities

1) A smooth sphere P, of mass 5 kg, moving with a speed of 2 m s^{-1} collides directly with a smooth sphere Q, of mass 3 kg, travelling in the opposite direction with a speed of 2 m s^{-1} on a smooth horizontal table. The coefficient of restitution for the collision is ½.

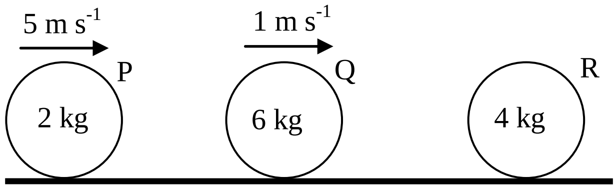


(i) Find the speeds of both spheres after this first collision.

As a result of the collision Q goes on to collide directly with a stationary smooth sphere R, of mass 4 kg. The collision between Q and R causes Q to come to rest.

(ii) Find the coefficient of restitution for the collision between Q and R.

2) A smooth sphere P, of mass 2 kg, moving with a speed of 5 m s^{-1} collides directly with a smooth sphere Q, of mass 6 kg, travelling in the same direction with a speed of 1 m s^{-1} on a smooth horizontal table. The coefficient of restitution for the collision is $\frac{2}{5}$.



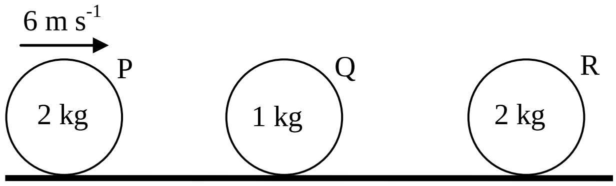
(i) Find the speeds of both spheres P and Q after this first collision.

As a result of the collision Q goes on to collide directly with a stationary smooth sphere R, of mass 4 kg. The collision between Q and R causes Q to lose 75% of its kinetic energy in the collision.

(ii) Find the speeds of both spheres Q and R after this second collision.

(iii) Find the coefficient of restitution for the collision between Q and R.

3) A smooth sphere P, of mass 2 kg, moving with a speed of 6 m s^{-1} collides directly with a smooth sphere Q, of mass 1 kg, which is at rest on a smooth horizontal table. The coefficient of restitution for the collision is ½.



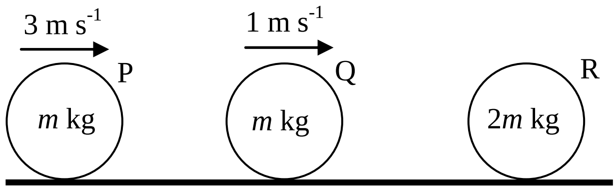
(i) Find the speed of both spheres after this first collision.

As a result of the collision Q goes on to collide directly with a stationary smooth sphere R, of mass 2 kg. The collision between Q and R causes Q to come to rest.

(ii) Find the coefficient of restitution for the collision between Q and R.

(iii) P then strikes Q again. Show that this is the final collision between the spheres.

4) A smooth sphere P, of mass *m* kg, moving with a speed of 3 m s^{-1} collides directly with a smooth sphere Q, also of mass *m* kg, travelling in the same direction with a speed of 1 m s^{-1} on a smooth horizontal table. After the collision P and Q keep moving in the same direction and the ratio of their speeds is: velocity of P : velocity of Q $= 3 : 5$.



(i) Find the speeds of both spheres after this first collision.

(ii) Find the coefficient of restitution for this collision.

As a result of the collision Q goes on to collide directly with a stationary smooth sphere R, of mass 2*m* kg. The collision between Q and R causes Q to lose 96% of its kinetic energy, but it continues to move in the same direction.

(iii) Find the speed of Q and the speed of R after this second collision.

(iv) Find the coefficient of restitution for the collision between Q and R.

Answers to Exercises

Exercise 5A

1) (i) 6 m s^{-1}, (ii) 0.8 m. 2) (i) 10 m s^{-1}, (ii) 3.2 m. 3) (i) 8 m s^{-1}, (ii) ¾. 4) (i) 12 m s^{-1}, (ii) ⅔.

Exercise 5B

1) 3 m s^{-1}, 10 m s^{-1}. 2) -1 m s^{-1}, 7 m s^{-1}. 3) 3 m s^{-1}, 6 m s^{-1}. 4) (i) 1, (ii) 2 m s^{-1}. 5) (i) 2, (ii) 3.6 m s^{-1}.
6) (ii) ½, (iii) 2.5 m s^{-1}. 7) (ii) ¾, (iii) 6.25 m s^{-1}.

Exercise 5C

1) (i) 0.9 m s^{-1}, 3.9 m s^{-1}, (ii) 4.35 J, (iii) 14.7 N s. 2) (i) 1 m s^{-1}, 2 m s^{-1}, (ii) ¾, (iii) 18 N s.
3) (i) 0.25 m s^{-1}, 4.75 m s^{-1}, (ii) 8.25 J, (iii) 16.5 N s. 4) (i) 3.5 m s^{-1}, 4 m s^{-1}, (ii) 12.75 J, (iii) 3 N s.
5) (i) 3.6 m s^{-1}, 5 m s^{-1}, (ii) 0.35, (iii) 15.6 J. 6) (i) $\frac{5}{16}$, (ii) 101.7 J. 7) (i) 4 m s^{-1}, (ii) ¼, (iii) 2.25 J, (iv) 3 N s.
8) (i) 1 m s^{-1}, (ii) ⅛, (iii) $\frac{21}{22}$. 9) (i) ⅔, (ii) ½.

Exercise 5D

1) (i) 0.25 m s^{-1}, 1.75 m s^{-1}, (ii) ¾. 2) (i) 0.8 m s^{-1}, 2.4 m s^{-1}, (ii) 1.2 m s^{-1}, 1.8 m s^{-1}, (iii) ¼.
3) (i) 3 m s^{-1}, 6 m s^{-1}, (ii) ½. 4) (i) 1.5 m s^{-1}, 2.5 m s^{-1}, (ii) ½, (iii) 0.5 m s^{-1}, 1 m s^{-1}, (iv) $\frac{1}{5}$.